Subleading corrections to parity violating pion photoproduction

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Abstract

We compute the photon asymmetry B_γ for near threshold parity violating (PV) pion photoproduction through sub-leading order. We show that sub-leading contributions involve a new combination of PV couplings not included in previous analyses of hadronic PV. We argue that existing constraints on the leading order contribution to B_γ – obtained from the PV γ-decay of ^{18}F – suggest that the impact of the subleading contributions may be more significant than expected from naturalness arguments.

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1 Introduction

The parity violating (PV) πNN Yukawa coupling constant h_π^{eff} is a key ingredient to the understanding of the PV nuclear interaction[1, 2, 3, 4, 5] (historically, this constant has been denoted as f_π in the literature). A number of hadronic PV experiments have sought to determine the value of h_π^{eff} [2, 5, 6, 7, 8, 9]. A particularly significant result has been obtained from measurements of photon polarization P_γ in the PV γ-decay of ^{18}F:

$$h_\pi^{\text{eff}} = (0.73 \pm 2.3)g_\pi \quad , \tag{1}$$

where $g_\pi = 3.8 \times 10^{-8}$ gives the scale of g_π in the absence of weak neutral currents[1]. An explicit SU(6)/quark model analysis[1], as well as "naturalness" arguments (see below), would suggest that h_π^{eff} should be closer to $10g_\pi$. The results of the ^{18}F measurement, which has been repeated by five different groups, is therefore suprising. The nature of the h_π^{eff} puzzle is further complicated by two additional observations:

- The governing PV mixing matrix element in ^{18}F can be related by isospin symmetry to two body component of the experimental rate for the analog β decay ^{18}Ne→^{18}F+e^+ + ν_e [10, 12]. Since $P_\gamma(^{18}$F$)$ is dominated by its sensitivity to h_π^{eff}, the bounds in Eq. (1) appear to be robust from the standpoint of many-body nuclear theory[2].

- A measurement by the Boulder group of the nuclear spin-dependent PV effects in 6S-7S transitions in the ^{133}Cs atom has been used in order to extract a value for the cesium nuclear anapole moment (AM)[8]. Recently, a full two-body calculation of the cesium AM

has been used to extract constraints on the long- and short-range components of the PV NN interaction [11]. When combined with the constraints on the short-range PV NN interaction, the cesium results imply a central value for h_π^{eff} of $\sim 10g_\pi$, in agreement with the "naturalness" estimate.

The status of h_π^{eff} may be clarified by a slate of new experiments – suggested, planned, or currently underway: $\vec{n}p \rightarrow d\gamma$ at LANSCE [13], $\gamma^*, \gamma d \rightarrow np$ at Jefferson Lab [14], the rotation of polarized neutrons in helium at NIST [9] as well as polarized Compton scattering processes [15, 16]. Since these processes involve one- and few-body systems, one anticipates new constraints on the PV NN interaction free from many-body uncertainties related to complex nuclei such as cesium or fluorine.

If the new experiments were to confirm the present ^{18}F constraints on h_π^{eff}, then one should attempt to understand the nucleon structure dynamics responsible for the reduction from its "natural" size. At the same time, it would become necessary to account for the sub-leading chiral structure of the PV πNN Yukawa interaction and its related observables. To that end, we recently computed the subleading chiral contributions to h_π^{eff}[17]. At leading order, h_π^{eff} is identical to the low-energy constant (LEC) h_π^1 appearing in the PV pion-nucleon chiral Lagrangian[4]. The subleading contributions, which vanish in the chiral limit, involve a host of new LEC's whose effect on h_π^{eff} is fortuitously enhanced. A similar set of LEC's appear in anapole moment contributions to the radiative corrections to backward angle PV ep scattering. These corrections, which have recently been determined by the SAMPLE collaboration [19], appear to be consider-

ably larger than one's theoretical expectation[18]. Thus, there appear to be several hints that the chiral expansion for hadronic PV may not behave as one naïvely expects.

With this situation in mind, we consider in this note the subleading chiral contributions to another PV observable: the polarization asymmetry B_γ for the charged pion photoproduction process

$$\overrightarrow{\gamma}(q^\mu; \epsilon^\mu) + p(P_i^\mu) \rightarrow \pi^+(k^\mu) + n(P_f^\mu) . \qquad (2)$$

which will be the focus of the proposed JLab study. Here, $q^\mu = (\omega, \mathbf{q})$, P_i^μ, $k^\mu = (\omega_\pi, \mathbf{k})$, and P_f^μ are the center-of-mass four-momenta of photon, proton, pion and neutron, respectively, and ϵ^μ is the photon polarization vector. The asymmetry B_γ, which arises from the interference of PV and parity conserving (PC) amplitudes, was first studied in the context of the conventional meson-exchange framework for hadronic PV in Refs. [23, 24]. Recently, Chen and Ji (CJ) proposed a measurement of B_γ at the Jefferson Lab and recast the earlier analyses in the context of heavy baryon chiral perturbation theory (HBCPT) [21, 22]. The authors emphasized that PV π photoproduction accesses the PV $NN\pi$ interaction directly, whereas in nuclear observables it is contained within the PV NN potential. For the threshold region, where all external momenta are well below the chiral symmetry breaking scale $\Lambda_\chi = 4\pi F_\pi \sim 1$ GeV, CJ obtain the "low-energy theorem" for the asymmetry:

$$B_\gamma(\omega_{\rm th}, \theta) = \frac{\sqrt{2}F_\pi(\mu_p - \mu_n)}{g_A m_N} h_\pi^1 . \qquad (3)$$

and the corrections from terms higher order in the chiral expansion were estimated to be around 20% [20]. The expression in Eq. (3) is consistent with the $h_\pi^{\rm eff}$ dominance of B_γ found in Ref. [24]. CJ also explored the kinematic behavior of B_γ, indicating that it could be large enough to be observed in a polarized photon beam experiment at Jefferson Laboratory.

In this paper, we show that inclusion of subleading contributions to the PV photoproduction amplitude leads to a chirally corrected low-energy theorem:

$$B_\gamma(\omega_{\rm th}, \theta) = \frac{\sqrt{2}F_\pi}{g_A m_N}\left[\mu_p - \mu_n(1 + \frac{m_\pi}{m_N})\right] h_\pi^1$$
$$- \frac{4\sqrt{2}m_\pi}{g_A \Lambda_\chi}\bar{C} , \qquad (4)$$

where the m_π/m_N represents the first recoil corrections to the leading order PV and PC photoproduction amplitudes and $\bar{C}$ is a new PV LEC defined below. In terms of chiral counting, the result of CJ appears at $\mathcal{O}(p^0)$ while the corrections arising in Eq. (4) occur at $\mathcal{O}(p)$. We note that the recoil and $\bar{C}$ terms shown explicitly in Eq. (4) constitute the complete set of subleading contributions to the PV photoproduction amplitude, since the effects of loops as well as pole diagrams involving decuplet intermediate states arise at $\mathcal{O}(p^2)$ and beyond.

At face value, the expression in Eq. (4) indicates that B_γ is governed by two, rather than one, PV LEC's—h_π^1 and $\bar{C}$, with associated kinematic factors of nearly equal magnitude. The actual situation, however, is more subtle. The naturalness arguments which imply h_π^1 should be $\sim 10g_\pi$ also lead one to expect $\bar{C} \sim g_\pi$. Thus, if these two LEC's were to have their natural size, the subleading contributions to B_γ would generate the anticipated 10% effect[1]. The results of the ^{18}F experiment, on the other hand, imply that h_π^1 is strongly suppressed from its natural scale. In this case, one would expect h_π^1 and $\bar{C}$ to be of comparable importance. Given the present lack of a first principle QCD calculation of these two LEC's, it is up to experiment to settle the question. As noted in CJ, if h_π^1 were to have its natural size, then a 20% determination of B_γ may be feasible at Jefferson Lab. On the other hand, a null result at this precision would be consistent with the ^{18}F experiment and would imply the need of additional measurements to separate h_π^1 and $\bar{C}$.

In the remainder of this paper, we discuss the calculations leading to our conclusions. In section 2, we summarize the formalism for treating hadronic PV in HBCPT. Section 3 gives the calculation of the subleading contributions to the PV photoproduction amplitude. In section 4, we discuss a field redefinition, first suggested in Ref. [29], which expresses the results of section 3 in a compact manner. In section 5, we consider the expected magnitudes of the PV LEC's, relate these estimates to the earlier work of Ref. [24], and summarize our conclusions.

2 Hadronic parity violation in chiral perturbation theory

Before considering the heavy baryon expansion, it is useful to review the relevant PC and PV Lagrangians in the fully relativistic theory. For simplicity, we consider only π, N, and γ interactions. In this case, for PC interactions one has

$$\begin{aligned}\mathcal{L}^{PC} &= \frac{1}{4}F_\pi^2 Tr D^\mu \Sigma D_\mu \Sigma^\dagger + \bar{N}(i\mathcal{D}_\mu \gamma^\mu - m_N)N \\ &+ g_A \bar{N}A_\mu \gamma^\mu \gamma_5 N + \frac{e}{\Lambda_\chi}\bar{N}(c_s \\ &+ c_v \tau_3)\sigma^{\mu\nu}N F_{\mu\nu}^+ + \cdots ,\end{aligned} \qquad (5)$$

where $\mathcal{D}_\mu$ is the chiral and electromagnetic (EM) covariant derivative, $\Sigma = \exp(i\vec{\tau} \cdot \vec{\pi}/F_\pi)$, N is the nucleon isodoublet field and

$$A_\mu = \frac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) \qquad (6)$$

$$F_{\mu\nu}^\pm = \frac{1}{2}F_{\mu\nu}(\xi \Lambda_p \xi^\dagger \pm \xi^\dagger \Lambda_p \xi) \qquad (7)$$

$$\Lambda_p = \frac{1}{2}(1 + \tau_3) . \qquad (8)$$

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[1]We thank the authors of CJ as well as J.L. Friar for clarification of this point.

The relevant PV Lagrangians are [4, 18]

$$
\begin{aligned}
\mathcal{L}^{PV} &= h_V^0 \bar{N} A_\mu \gamma^\mu N + \frac{h_V^1}{2} \bar{N} \gamma^\mu N Tr(A_\mu X_+^3) \\
&\quad - \frac{h_A^1}{2} \bar{N} \gamma^\mu \gamma_5 N Tr(A_\mu X_-^3) - \frac{h_\pi^1}{2\sqrt{2}} F_\pi \bar{N} X_-^3 N \\
&\quad + h_V^2 \mathcal{I}^{ab} \bar{N} [X_R^a A_\mu X_R^b + X_L^a A_\mu X_L^b] \gamma^\mu N \\
&\quad - \frac{h_A^2}{2} \mathcal{I}^{ab} \bar{N} [X_R^a A_\mu X_R^b - X_L^a A_\mu X_L^b] \gamma^\mu \gamma_5 N \\
&\quad + \frac{c_1}{\Lambda_\chi} \bar{N} \sigma^{\mu\nu} [F_{\mu\nu}^+, X_-^3]_+ N + \frac{c_2}{\Lambda_\chi} \bar{N} \sigma^{\mu\nu} F_{\mu\nu}^- N \\
&\quad + \frac{c_3}{\Lambda_\chi} \bar{N} \sigma^{\mu\nu} [F_{\mu\nu}^-, X_+^3]_+ N \; ,
\end{aligned}
\tag{9}
$$

where

$$
\begin{aligned}
X_L^a &= \xi^\dagger \tau^a \xi \tag{10} \\
X_R^a &= \xi \tau^a \xi^\dagger \tag{11} \\
X_\pm^a &= X_L^a \pm X_R^a \tag{12}
\end{aligned}
$$

and where we follow the sign convention of Ref. [18, 17]. The corresponding PC and PV Lagrangians involving Δ fields are given in Ref. [18].

Of the PV LEC's appearing in Eqs. (9), h_π^1 is the most familiar and has received the most extensive theoretical scrutiny[1, 2, 3, 4, 5]. In the context of chiral perturbation theory, the radiative corrections to h_π^1 were discussed extensively in [17], where it was pointed out that what nuclear PV experiments measure is an effective coupling h_π^{eff} [17], which is a linear combination of LECs $h_\pi^1, h_\Delta, h_A^{(i)}$ etc. The commonly used "best value"— $|h_\pi^1| = 5 \times 10^{-7}$—quoted in [1] corresponds to a large extent to a simple tree-level estimate without loop corrections. Estimates for h_V^i and h_A^i have been discussed in Refs. [4, 17], though no analysis similar to that of [1] has been performed. To date, there have appeared no estimates of the PV $NN\pi\gamma$ constants c_i. Nevertheless, one expects the magnitude of these LEC's to be roughly a few times g_π.

For purposes of computing B_γ, it is necessary to expand the non-linear Lagrangians of Eqs. (5,9) through one π and one γ order. The results for the PC interactions are familiar and we do not list them here. For the PV Lagrangians, we also include the leading (2π) terms proportional to h_A^i—

$$
\begin{aligned}
\mathcal{L}^{PV} &= -ih_\pi^1 \pi^+ p^\dagger n \\
&\quad - \frac{h_V}{\sqrt{2} F_\pi} \bar{p} \gamma^\mu n D_\mu \pi^+ \\
&\quad + i \frac{h_A^{(1)} + \overline{h}_A^{(2)}}{F_\pi^2} \bar{p} \gamma^\mu \gamma_5 p \pi^+ D_\mu \pi^- \\
&\quad + i \frac{h_A^{(1)} - \overline{h}_A^{(2)}}{F_\pi^2} \bar{n} \gamma^\mu \gamma_5 n \pi^+ D_\mu \pi^- \\
&\quad - ie \frac{C}{\Lambda_\chi F_\pi} \bar{p} \sigma^{\mu\nu} F_{\mu\nu} n \pi^+ + \text{h.c.} \quad , \tag{13}
\end{aligned}
$$

where

$$
\begin{aligned}
h_V &= h_V^0 + \tfrac{4}{3} h_V^2 \\
C &= -2\sqrt{2} c_1 + \tfrac{1}{\sqrt{2}} c_2 \quad .
\end{aligned}
\tag{14}
$$

Note that the LEC h_V^1 does not contribute to $\mathcal{L}^{PV}$ at this order. As noted in Ref. [29] and discussed in detail below, the effects of the h_V^i Lagrangians on processes involving up to two pions and one photon can be absorbed into effective C- and h_A^i type Lagrangians through 2π order via an appropriate nucleon field redefinition. The reason is that when one integrates by parts the action corresponding to the h_V-term in Eq. (13), the integrand vanishes by the nucleon equations of motion. At 3π order and beyond, however, the effects of the h_V^i terms in Eq. (9) cannot be absorbed into other effective interactions via field redefinition. Thus, in the context of the complete non-linear PV Lagrangian, the h_V^i remain distinct LEC's. Consequently, we keep the h_V-dependence explicit in what follows.

3 The subleading correction to the asymmetry

In order to maintain proper chiral counting, we use the heavy baryon expansion of Eqs. (5,9). The motivation behind the use of heavy baryon chiral perturbation theory (HBCPT) is explained in detail in [20], and we follow the notations of this reference. Since we work in the near-threshold region, we use the so-called "small-scale" expansion [25], i.e., we treat $\omega, \omega_\pi, |k|, m_\pi, \delta = m_\Delta - m_N$, etc. as small quantities and characterize amplitudes by the number of powers of these terms, e.g., we count the term $\omega_\pi / q \cdot k$ as being $\mathcal{O}(p^{-1})$. The photon asymmetry arises from the interference of the parity conserving (PC) and PV amplitudes. In Ref. [20] the asymmetry was truncated at leading order, i.e., $\mathcal{O}(p^0)$. In the present work we include the $\mathcal{O}(p)$ correction, which arises dominantly from the PV vector πNN couplings. As we show below, chiral loops contribute to the asymmetry only at $\mathcal{O}(p^2)$ and higher. Hence, our truncation of the chiral expansion of the asymmetry is consistent and complete up to terms of $\mathcal{O}(p)$.

The PC amplitudes which describe the charged photoproduction reaction are defined via

$$
\begin{aligned}
T^{PC} &= N^\dagger \Big[i\mathcal{A}_1 \vec{\sigma} \cdot \hat{\epsilon} + i\mathcal{A}_2 \vec{\sigma} \cdot \hat{\mathbf{q}} \; \hat{\epsilon} \cdot \hat{\mathbf{k}} \\
&\quad + i\mathcal{A}_3 \vec{\sigma} \cdot \hat{\mathbf{k}} \; \hat{\epsilon} \cdot \hat{\mathbf{k}} + \mathcal{A}_4 \hat{\epsilon} \cdot \hat{\mathbf{q}} \times \hat{\mathbf{k}} \Big] N \; , \tag{15}
\end{aligned}
$$

where N is the proton Pauli spinor, $\vec{\sigma}$ are the Pauli spin matrices, and $\hat{\mathbf{q}}$ and $\hat{\mathbf{k}}$ are the unit vectors in the photon and pion directions respectively. At leading order in HBCPT, we have $\mathcal{A}_1 = eg_A/\sqrt{2}F_\pi$, $\mathcal{A}_2 = \mathcal{A}_1 \omega |\mathbf{k}|/q \cdot k$, $\mathcal{A}_3 = -\mathcal{A}_1 \mathbf{k}^2/q \cdot k$, and $\mathcal{A}_4 = 0$ [26, 27]. As explained in [20] one also requires the non-vanishing subleading order

result for $\mathcal{A}_4$.

$$\mathcal{A}_4 = \frac{eg_A|\mathbf{k}|}{2\sqrt{2}F_\pi m_N}\left[\mu_p - \left(\frac{\omega}{\omega_\pi}\right)\mu_n\right]$$
$$- \frac{2eg_{\pi N\Delta}G_1|\mathbf{k}|}{9\sqrt{2}F_\pi m_N}\left(\frac{\omega}{\omega-\delta} + \frac{\omega}{\omega_\pi+\delta}\right) \quad , \quad (16)$$

where the $\Delta(1232)$ contribution has been included explicitly. Here G_1 is the M1 transition moment connecting the nucleon and delta, and $g_{\pi N\Delta}$ is the π-N-Δ coupling[25]. Note that $\mathcal{A}_{1-3}$ is $\mathcal{O}(p^0)$ while $\mathcal{A}_4$ is $\mathcal{O}(p)$.

To $\mathcal{O}(p)$ in the chiral expansion, the PV $\gamma p \to \pi^+ n$ T-matrix can be written as

$$T^{PV} = N^\dagger\left[\mathcal{F}_1\widehat{\mathbf{k}}\cdot\hat{\epsilon} + i\mathcal{F}_2\vec{\sigma}\cdot\hat{\epsilon}\times\widehat{\mathbf{q}} + i\mathcal{F}_3\vec{\sigma}\cdot\hat{\epsilon}\times\widehat{\mathbf{k}}\right]N \ . \tag{17}$$

We then have the asymmetry

$$B_\gamma \sim \{\mathcal{A}_1\mathcal{F}_2 + \frac{\sin^2\theta}{2}[\mathcal{A}_3\mathcal{F}_2 - A_4\mathcal{F}_1 - \mathcal{A}_2\mathcal{F}_3] + \cos\theta\mathcal{A}_1\mathcal{F}_3\} \ , \tag{18}$$

where $\theta = \cos^{-1}\hat{\mathbf{q}}\cdot\hat{\mathbf{k}}$. (Note that the nominally leading piece from the interference term $\mathcal{A}_{1-3}\mathcal{F}_1$ vanishes if the proton target is unpolarized.)

The leading, nonvanishing contributions to B_γ, which occur at $\mathcal{O}(p^0)$, are generated by the $\mathcal{O}(p^0)$ terms in $\mathcal{A}_{1-3}$ interfering with the $\mathcal{O}(p^0)$ terms in $\mathcal{F}_2$, and by the $\mathcal{O}(p)$ term in $\mathcal{A}_4$ interfering with the $\mathcal{O}(p^{-1})$ term in $\mathcal{F}_1$. The leading order PV contributions to $\mathcal{F}_{1,2}$ arise from the insertion of the PV Yukawa πNN vertex of Eq. (13) in FIG. 1 (a), (c), (d). The results, given in Ref. [20], are

$$\mathcal{F}_1 = -\frac{eh_\pi^1|\mathbf{k}|}{q\cdot k} \ , \ \mathcal{F}_2 = -\frac{eh_\pi^1}{2m_N}\left[\mu_p - \left(\frac{\omega}{\omega_\pi}\right)\mu_n\right] \ . \tag{19}$$

where $\mathcal{F}_1, \mathcal{F}_2$ are $\mathcal{O}(p^{-1}), \mathcal{O}(p^0)$ respectively.

Subleading contributions to B_γ are generated by $\mathcal{O}(p)$ and $\mathcal{O}(p^2)$ terms in $\mathcal{A}_{1-3}$ and $\mathcal{A}_4$, respectively, interfering with the amplitudes in Eq. (19), and by $\mathcal{O}(p)$ contributions in $\mathcal{F}_{2,3}$ interfering with the $\mathcal{O}(p^0)$ terms in $\mathcal{A}_{1-3}$. The subleading PC contributions have been computed in [27]. We refer to the detailed expressions for these corrections in that work, which we employ in our numerical analysis below. Of greater interest are the $\mathcal{O}(p)$ PV amplitudes involving new LEC's. These contributions, which are generated by the h_V and C terms in eq. (13), contribute to both the pole diagrams FIG. 1 (c), (d) and the seagull diagram FIG. 1 (b). We have

$$\mathcal{F}_1 = \mathcal{F}_3 = 0 \ ,$$
$$\mathcal{F}_2 = \frac{eh_V}{2m_N}\frac{\omega_\pi}{\sqrt{2}F_\pi}\left[\mu_p - \frac{\omega}{\omega_\pi}\mu_n - \frac{\omega}{\omega_\pi}\right] + \frac{2eC}{\Lambda_\chi}\frac{\omega}{F_\pi} \ . \tag{20}$$

The contribution from FIG. 1 (b) cancels exactly those from FIG. 1 (c), (d) where the γNN vertex is minimum coupling [2].

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[2] We thank J.-W. Chen and X. Ji for pointing out this cancellation to us.

According to the expression in Eq. (20), the h_V and C contributions to $\mathcal{F}_2$ carry distinct kinematic dependences, a feature which might suggest using the ω-dependence of B_γ to separate the two LEC's. Such a program would be misguided, however. As we show below, the kinematic behavior generated by the h_V and C interactions is identical when a fully relativistic framework is used to compute the PV amplitudes. The result in this case is

$$\mathcal{F}_2 = \frac{2e\bar{C}}{\Lambda_\chi}\frac{\omega}{F_\pi} \tag{21}$$

with

$$\bar{C} = C + \frac{\Lambda_\chi}{m_N}\left(\frac{\kappa_p - \kappa_n}{4\sqrt{2}}\right)h_V \quad . \tag{22}$$

Here κ_i are the anomalous nucleon magnetic moments, as distinguished from the full moments μ_i used to this point. The apparent difference between Eqs. (20,21) is a artifact of truncating the $1/m_N$ expansion at this order in HBCPT—to this order of the chiral expansion the photon and pion energies are equal. In what follows, then, we adopt the result in Eqs. (21).

In addition to the $\mathcal{O}(p)$ contributions from h_V and C, $\mathcal{F}_2$ receives an $\mathcal{O}(p)$ contribution involving h_π^1 generated by the $1/m_N$ corrections to the nucleon propagator and γNN vertex in the pole amplitudes. We include these corrections in the asymmtry formulae below. Other possible contributions to the PV amplitudes include PV $\gamma N\Delta$ and $\pi N\Delta$ interactions. However, both contribute at $\mathcal{O}(p^2)$, which is higher than the order at which we are truncating. Similarly chiral loop contributions to $\mathcal{A}_{1-4}, \mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3$ appear at $\mathcal{O}(p^2)$, $\mathcal{O}(p)$, $\mathcal{O}(p^2)$ or higher, respectively. Consequently, chiral loops do not contribute to the asymmetry until at least $\mathcal{O}(p^2)$. All such contributions are then higher order and can be dropped.

The resultant photon asymmetry at order $\mathcal{O}(p)$ reads then

$$B_\gamma(\omega,\theta) = \frac{\sqrt{2}h_\pi^1 F_\pi}{g_A m_N \mathcal{G}}\left\{\left[\left(1 - \frac{\omega}{2m_N}\right)\mu_p\right.\right.$$
$$\left.- \left(\frac{\omega}{\omega_\pi}\right)\left(1 - \frac{|\mathbf{k}|^2}{2m_N\omega_\pi}\right)\mu_n\right]\left(1 - \sin^2\theta\frac{\mathbf{k}^2}{q\cdot k}\right)$$
$$+ \frac{2}{9}g_{\pi N\Delta}G_1\sin^2\theta\frac{\mathbf{k}^2}{g_A q\cdot k}\left(\frac{\omega}{\omega-\delta} + \frac{\omega}{\omega_\pi+\delta}\right)$$
$$\left.+ 2\left(\frac{\omega}{\omega_\pi}\right)\frac{|\mathbf{k}|}{m_N}\mu_n\left(\cos\theta - \sin^2\theta\frac{\omega|\mathbf{k}|}{2q\cdot k}\right)\right\}$$
$$- \frac{4\sqrt{2}\bar{C}}{g_A\Lambda_\chi\mathcal{G}}\left(1 - \sin^2\theta\frac{\mathbf{k}^2}{2q\cdot k}\right) + \cdots \quad . \tag{23}$$

where the ellipses indicate the PC $1/m_N$ contributions of T^{PC} in Ref. [27] and

$$\mathcal{G} = 1 - \sin^2\theta\frac{\mathbf{k}^2}{q\cdot k}[1 - \frac{(\mathbf{q}-\mathbf{k})^2}{2q\cdot k}] \ . \tag{24}$$

At threshold—$|\mathbf{k}| = 0$—Eq. (23) becomes the low energy theorem for the photon asymmetry given in Eq. (4).

4 Field redefinition and physical observables

In response to an earlier version of this paper, CJ observed that one may obtain the subleading PV contributions to B_γ involving $\bar{C}$ entirely from the diagram (b) in Fig. 1 after a suitable redefinition of the nucleon fields [29]. This simplification arises because the h_V-terms in Eq. (13) vanish for on-shell nucleons after integration by parts. As discussed in Ref. [30], the effects of interactions which vanish by the equations of motion can always be absorbed into contact interactions via field redefinition. In the present case, the redefinition proposed by CJ is

$$p = \tilde{p} - \frac{i}{\sqrt{2}F_\pi}h_V\pi^+\tilde{n}\,,$$
$$n = \tilde{n} - \frac{i}{\sqrt{2}F_\pi}h_V\pi^-\tilde{p}\,. \quad (25)$$

The resultant PV Lagrangian $\tilde{\mathcal{L}}^{PV}$ is

$$\begin{aligned} \tilde{\mathcal{L}}^{PV} &= -ih_\pi^1\pi^+\bar{\tilde{p}}\tilde{n} \\ &+ i\frac{h_A^1 + \overline{h}_A^2}{F_\pi^2}\bar{\tilde{p}}\gamma^\mu\gamma_5\tilde{p}\pi^+ D_\mu\pi^- \\ &+ i\frac{h_A^1 - \overline{h}_A^2}{F_\pi^2}\bar{\tilde{n}}\gamma^\mu\gamma_5\tilde{n}\pi^+ D_\mu\pi^- \\ &- ie\frac{\overline{C}}{\Lambda_\chi F_\pi}\bar{\tilde{p}}\sigma^{\mu\nu}F_{\mu\nu}\tilde{n}\pi^+ + \text{h.c.} + \cdots \,. \end{aligned} \quad (26)$$

where

$$\overline{h}_A^{(2)} = h_A^{(2)} - \frac{g_A}{2}h_V\,, \quad \overline{C} = C + \frac{\Lambda_\chi}{m_N}\left(\frac{\kappa_p - \kappa_n}{4\sqrt{2}}\right)h_V\,. \quad (27)$$

Note that in $\tilde{\mathcal{L}}^{PV}$, the h_V-terms have been eliminated, and their effect absorbed into the LEC $\bar{C}$ and $\overline{h}_A^{(2)}$ introduced earlier[3]. In terms of physical observables involving up to two π and one γ, it is not possible to determine h_V from C. In particular, as noted in Ref. [18], the PV NN potential contains no dependence on h_V.

The question remains as to whether the h_V^i constitute distinct LEC's in the context of the full nonlinear Lagrangian of Eq. (9), or whether their effects can be entirely absorbed into other LEC's. In the following, we address this question using the simplest unitarized version of the transformation in Eq. (25). We show that at 3π order, it is not possible to eliminate the h_V^i effects in terms of other LEC's. We give a general proof of this result in the Appendix. In principle, then, one could use an appropriate PV 3π process (e.g., the analyzing power for $\pi^-\vec{p} \to \pi^+\pi^- n$) to separate the h_V^i and C. In practice, measurements of multi-pion processes would be extremely difficult at best.

To illustrate this result, consider the unitary transformation

$$N = V_1\tilde{N} \quad (28)$$

to eliminate the leading linear term after expansion of PV vector pieces in (9). The explicit expression of V_1 is

$$V_1 = e^{-\frac{i}{F_\pi}\hat{O}_1} = Ve^{-\frac{ih_V^1}{F_\pi}\pi^0} \quad (29)$$

$$V = e^{-\frac{i}{F_\pi}\hat{O}} \quad (30)$$

$$\hat{O}_1 = \hat{O} + h_V^1\pi^0\hat{1} \quad (31)$$

$$\hat{O} = \frac{h_V^0}{2}\pi^i\tau^i + \frac{4}{3}h_V^2\begin{pmatrix} 2\pi^0 & \frac{\pi^+}{\sqrt{2}} \\ \frac{\pi^-}{\sqrt{2}} & -2\pi^0 \end{pmatrix} \quad (32)$$

The difference between the field redefinition Eq. (25) and Eq. (28) is two-fold. The latter is unitary and also takes into account the PV vector $\pi^0 NN$ interaction.

It is useful to collect some relevant terms of the redefined Lagrangians containing the nucleon field $\tilde{N}$. For the strong and electromagnetic part we have

$$\begin{aligned} \tilde{\mathcal{L}}^{PC} &= \bar{\tilde{N}}(iD_\mu\gamma^\mu - m_N)\tilde{N} + \bar{\tilde{N}}[V_1^\dagger iD_\mu V_1]\gamma^\mu\tilde{N} \\ &+ \bar{\tilde{N}}[V^\dagger iV_\mu V]\gamma^\mu\tilde{N} + g_A\bar{\tilde{N}}[V^\dagger A_\mu V]\gamma^\mu\gamma_5\tilde{N} \\ &+ \frac{e}{\Lambda_\chi}\bar{\tilde{N}}[V^\dagger(c_s + c_v\tau_3)\sigma^{\mu\nu}F_{\mu\nu}^+V]\tilde{N} + \cdots \end{aligned} \quad (33)$$

where V_μ is the chiral connection.

For the originally weak interaction we have

$$\begin{aligned} \tilde{\mathcal{L}}^{PV} &= h_V^0\bar{\tilde{N}}V^\dagger A_\mu V\gamma^\mu\tilde{N} + \frac{1}{2}h_V^1\bar{\tilde{N}}\gamma^\mu\tilde{N}Tr(A_\mu X_+^3) \\ &- \frac{1}{2}h_A^1\bar{\tilde{N}}\gamma^\mu\gamma_5\tilde{N}Tr(A_\mu X_-^3) - \frac{1}{2\sqrt{2}}h_\pi^1 F_\pi\bar{\tilde{N}}[V^\dagger X_-^3 V]\tilde{N} \\ &+ h_V^2\mathcal{I}^{ab}\bar{\tilde{N}}V^\dagger[X_R^a A_\mu X_R^b + X_L^a A_\mu X_L^b]V\gamma^\mu\tilde{N} \\ &- \frac{1}{2}h_A^2\mathcal{I}^{ab}\bar{\tilde{N}}V^\dagger[X_R^a A_\mu X_R^b - X_L^a A_\mu X_L^b]V\gamma^\mu\gamma_5\tilde{N} \\ &+ \frac{1}{\Lambda_\chi}c_1\bar{\tilde{N}}\sigma^{\mu\nu}[F_{\mu\nu}^+, X_-^3]_+\tilde{N} + \frac{1}{\Lambda_\chi}c_2\bar{\tilde{N}}\sigma^{\mu\nu}F_{\mu\nu}^-\tilde{N} \\ &+ \frac{1}{\Lambda_\chi}c_3\bar{\tilde{N}}\sigma^{\mu\nu}[F_{\mu\nu}^-, X_+^3]_+\tilde{N}\,, \end{aligned} \quad (34)$$

Now expand Eqs. (33) and (34) in $1/F_\pi$. The leading term arising from

$$\bar{\tilde{N}}[V_1^\dagger iD_\mu V_1]\gamma^\mu\tilde{N} \quad (35)$$

in Eq. (33) entirely cancels the 1π h_V^i terms in Eq. (34), recovering the results of Eqs. (26-27). The potential sources of 3π PV interactions include the following:

(1) expansion of the term in Eq. (33) $\bar{\tilde{N}}[V_1^\dagger iD_\mu V_1]\gamma^\mu\tilde{N}$ in Eq. (33). The result is $\mathcal{O}(G_F^3)$.

(2) expansion of the term in Eq. (33) $\bar{\tilde{N}}[V^\dagger iV_\mu V]\gamma^\mu\tilde{N}$, which is linear in h_V^i, $i = 0, 2$ only [$\mathcal{O}(G_F)$];

(3) expansion of $A_\mu, X_\pm^3, X_{L,R}^a$ operators in Eq.(34) to third order, which is linear in h_V^i, ($i = 0, 1, 2$) and h_A^i, h_π^1 ($i = 1, 2$) [$\mathcal{O}(G_F)$];

(4) expansion of V and $V^\dagger$ operator in Eq.(34) to second order, which is cubic in h_V^i, $i = 0, 2$ only [$\mathcal{O}(G_F^3)$].

(5) expansion of the $\bar{\tilde{N}}[V^\dagger A_\mu V]\gamma^\mu\gamma^5\tilde{N}$ and $\bar{\tilde{N}}[V^\dagger(c_s + c_v\tau_3)\sigma^{\mu\nu}F_{\mu\nu}^+V]\tilde{N}$ terms in to third order [$\mathcal{O}(G_F^2)$ and $\mathcal{O}(G_F)$, respectively].

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[3]Our relative phase between C and h_V in $\bar{C}$ differs from Ref. [29].

(6) expansion of the c_i-terms in Eq. (34) to third order [$\mathcal{O}(G_F)$].

Prior to the applying the transformation (28,29), the only PV $NN\pi\pi$ contact interactions arise from the h_V^i-terms in (3). After field redefinition, one must add up all six contributions. Note that those arising from (5), (6) and the h_π^1, h_A^i-terms in (3) contain a different Lorentz structure than the h_V^i terms in (3) and therefore cannot cancel the latter. Similarly, since the h_V^i 3π terms in (3) arise at $\mathcal{O}(G_F)$, they cannot be cancelled by the contributions from (1) and (4). Thus, at $\mathcal{O}(G_F)$, the only 3π contributions involving $\tilde{\bar{N}}\gamma_\mu\tilde{N}$ arise from (2) and the h_V^i terms in (3). Note that (2) contains no terms involving h_V^1. Hence, the 3π term proportional to h_V^1 appearing in (3) cannot be removed by the transformation Eq. (28).

For the terms proportional to h_V^0 we obtain from (2)

$$-\frac{h_V^0}{2F_\pi^3}[\pi,[\pi,D_\mu\pi]] \ . \tag{36}$$

where $\pi = \frac{1}{2}\pi^i\tau^i$, while (3) yields

$$+\frac{h_V^0}{6F_\pi^3}[\pi,[\pi,D_\mu\pi]] \ . \tag{37}$$

Their sum is

$$-\frac{h_V^0}{3F_\pi^3}[\pi,[\pi,D_\mu\pi]] \ . \tag{38}$$

The 3π PV vector h_V^0 contact term does not vanish after field redefinition. A similar result holds for h_V^2.

As we show in the Appendix, one may remove the 1π h_V^i terms by a more general field redefinition than given by Eqs. (28,29). Nevertheless, it is still not possible to remove the 3π terms proportional to the h_V^i (the arguments of the proof are similar to those above, but more tedious in the details). Thus, we conclude that the h_V^i constitute distinct and, in principle measurable LEC's in the nonlinear chiral theory of Eqs. (5,9). While one could compute observables in either formulation of the theory (with or without the field redefinition) and obtain identical results, the structure of Lagrangian is more cumbersome after application of Eq. (28): there appear several new interaction vertices, including small [$\mathcal{O}(G_F^2)$] parity-conserving non-derivative interactions; the chiral transformation properties are less transparent than in the original version of the theory; and the fields $\tilde{N}$ annihilate nucleon states of mixed parity. Consequently, we retain the original form of $\mathcal{L}^{PV}$ given in Refs. [4, 18].

5 Scale of the LEC's

Given that h_π^1 and $\bar{C}$ appear in B_γ with nearly equal weight, it would be useful to have in hand a theoretical expectation for the magnitudes of these LEC's. A simple estimate can be obtained by applying the "naïve dimensional analysis" of Ref. [31]. For strong and EM interations, effective interactions scale with F_π and Λ_χ as

$$(\Lambda_\chi F_\pi)^2 \ \times \ \left(\frac{\bar{N}N}{\Lambda_\chi F_\pi^2}\right)^k \left(\frac{\pi}{F_\pi}\right)^l \left(\frac{D_\mu}{\Lambda_\chi}\right)^m \ , \tag{39}$$

where k,l,m are integers and D_μ is the covariant derivative. For weak interactions, the same counting applies, multiplied by an overall scale of

$$g_\pi \sim \frac{G_F F_\pi^2}{2\sqrt{2}} \ . \tag{40}$$

Thus, one would expect the strength of the PV $NN\pi$ Yukawa interaction to be given by Eqs. (39,40) with $k = 1$, $l = 1$, $n = 0$:

$$\frac{\Lambda_\chi}{F_\pi} \ g_\pi = 4\pi g_\pi \ . \tag{41}$$

Since the definition of the Yukawa interaction in Eq. (9) contains no explicit factors of Λ or F_π, one expects the natural size of this LEC to be given by Eq. (41). Similarly, the $\bar{C}$ interaction, which involves $k = 1$, $l = 1$, $m = 2$, should scale as

$$\frac{1}{\Lambda_\chi F_\pi} \ g_\pi \ . \tag{42}$$

However, since the PV $NN\pi\gamma$ contact interaction in Eq. (34) already contains the explicit factors $1/\Lambda_\chi$ and $1/F_\pi$, the coefficient $-\bar{C}$ should be roughly of size g_π.

It is useful to compare these expectations with results of model calculations as well as with experiment. The benchmark SU(6)/quark model calculation of Ref. [1], updated in Ref. [3], gives a "best" estimate for h_π^{eff} of $(7-12)\times g_\pi$ – roughly commensurate with the expection of Eq. (41). That analysis, however, allows for the Yukawa coupling to be as small as zero and as large as $(20-30)\times g_\pi$, owing to uncertainties associated with various SU(6) reduced matrix elements and quark model inputs. To date, no estimate of $\bar{C}$ has been performed. A simple estimate can be made, however, by assuming the short-distance PV physics is saturated by t-channel vector meson exchange. In the purely mesonic sector, one may understand the magnitudes of the $\mathcal{O}(p^4)$ LEC's L_i using vector meson saturation. For the baryon sector, the same framework was used to estimate the sub-leading contributions to the nucleon anapole moment [18]. In the present instance, an illustrative contribution in this context is given in Fig. 2, where the $\bar{C}$-amplitude is generated by the PV ρNN interaction. For the $\rho\pi\gamma$ vertex we use the Lagrangian:

$$\mathcal{L}_{\rho\pi\gamma}^{PC} = e\frac{g_{\rho\gamma}}{4m_\rho}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}G_{\alpha\beta}^-\pi^+ + \cdots \tag{43}$$

where $G_{\alpha\beta} = \partial_\alpha\rho_\beta - \partial_\beta\rho_\alpha$. From the ρ radiative decay width [28] we have $|g_{\rho\gamma}| = 0.6$, and for the PV ρNN interaction we follow Ref. [1], writing

$$\mathcal{L}_{\rho NN}^{PV} = \sqrt{2}(h_\rho^0 - \frac{h_\rho^2}{2\sqrt{6}})[\bar{p}\gamma_\mu\gamma_5\rho^+n + H.c. + \cdots] \tag{44}$$

Invoking VMD we have

$$\begin{aligned}
\bar{C} &\sim -\frac{g_{\rho\pi\gamma}}{\sqrt{2}}\frac{\Lambda_\chi F_\pi m_\pi}{m_\rho^3}(h_\rho^0 - \frac{h_\rho^2}{2\sqrt{6}}) \\
&\sim -0.35 g_\pi \quad ,
\end{aligned} \tag{45}$$

where we have used the DDH "best values" $h_\rho^0 = -30 g_\pi, h_\rho^2 = -25 g_\pi$ [1]. Presumably, other heavy mesons contribute with comparable strength. In this simple vector meson saturation picture, then, the size of $\bar{C}$ is consistent with the expectation in Eq. (42). We note that the authors of Ref. [24] adopted similar picture for the short-distance PV physics, treating the ρ and ω as explicit dynamical degrees of freedom.

As stated at the outset of this work, the quandry for the effective field theory treatment of B_γ is that the constraints on h_π^1 from the $P_\gamma({}^{18}\text{F})$ measurements imply that this coupling is considerably suppressed from its "natural" scale[4]. While the analysis of Refs. [1, 3] can accomodate the ${}^{18}\text{F}$ result, one has a more difficult task of explaining this result using effective field theory alone, without reference to the underlying dynamics of strong and weak interactions. Nevertheless, taking the ${}^{18}\text{F}$ result at face value implies that in the HBCPT treatment of one- and few-body PV processes nominally sensitive to the PV πNN Yukawa coupling, one must also take into consideration subleading PV contributions as we have done for B_γ. Disentangling the short-distance physics responsible for these subleading effects then remains an interesting and unsolved problem for both theory and experiment.

Acknowledgment

6 Appendix A

We present here a general proof that the PV 3π vector interaction vertex (proportional to the h_V^i) cannot be removed by any unitary transformation U. To simplify notation, we absorb the factor $1/F_\pi$ into the pion field. From now on it is understood that

$$\pi = \frac{1}{F_\pi}\pi^i\frac{\tau^i}{2} \quad . \tag{46}$$

Since the transformation is unitary, we have

$$\hat{U} = e^{-i\hat{F}} \tag{47}$$

$$\hat{F} = \hat{F}^\dagger \tag{48}$$

The operator $\hat{F}$ can be expanded in terms of the number of pions. Since $\hat{F}$ should not carry explicit Lorentz indices, any derivatives should appear in pairs. Because we are discussing 3π PV vertex with only one derivative in the present case, the possible derivative terms are irrelevant here. Consequently, we omit them from the following discussion. We also consider explicitly only the $h_V^{0,1}$ contributions; the arguments involving h_V^2 are similar, but considerably more tedious.

Expand $\hat{F}$:

$$\hat{F} = \hat{O}_1 + \hat{O}_2 + \hat{O}_3 + \cdots \quad , \tag{49}$$

where $\hat{O}_n$ contains products of $n\pi$ fields. The leading term $\hat{O}_1$ is needed to remove the 1π PV vector linear term. Its structure is fixed and of $\mathcal{O}(G_F)$ as discussed in Section 4. The remaining terms $\hat{O}_n$, $n > 1$ could, in principle, be of $\mathcal{O}(G_F^0)$. In the present case, we need to consider only the terms through $n = 3$. The most general forms of $\hat{O}_2, \hat{O}_3$ read

$$\begin{aligned}
\hat{O}_2 = &(a_1\pi^+\pi^- + a_2\pi^0\pi^0)\hat{1} \\
&+(a_3\pi^+\pi^- + a_4\pi^0\pi^0)\tau_3 \\
&+a_5\pi^0(\pi^+\tau_+ + \pi^-\tau_-) \\
&+ia_6\pi^0(\pi^+\tau_+ - \pi^-\tau_-)
\end{aligned} \tag{50}$$

$$\begin{aligned}
\hat{O}_3 = &(b_1\pi^+\pi^- + b_2\pi^0\pi^0)\pi^0\hat{1} \\
&+(b_3\pi^+\pi^- + b_4\pi^0\pi^0)\pi^0\tau_3 \\
&+(b_5\pi^+\pi^- + b_6\pi^0\pi^0)(\pi^+\tau_+ + \pi^-\tau_-) \\
&+i(b_7\pi^+\pi^- + b_8\pi^0\pi^0)(\pi^+\tau_+ - \pi^-\tau_-)
\end{aligned} \tag{51}$$

where a_{1-6}, b_{1-8} are real numbers.

Now perform the unitary transformation

$$N = \hat{U}\tilde{N} \quad . \tag{52}$$

The possible sources of PV vector 3π vertices in the transformed Lagrangians are the same as discussed in the section 4 (items 1-6, but the order in G_F is not *a priori* fixed here). In addition, we must also expand the $X_\pm^a$ along with A_μ in item (3). As was done previously, we may neglect those terms whose Lorentz structure differs from $\tilde{N}\gamma_\mu\tilde{N}$. Thus, we consider only the vector terms arising from (1)-(4) (with $V \to \hat{U}$). From (1) we obtain the three π contribution

$$\hat{U}^\dagger iD_\mu\hat{U} = D_\mu\hat{O}_3 + i[\hat{O}_1, D_\mu\hat{O}_2] - i[D_\mu\hat{O}_1, \hat{O}_2] + \mathcal{O}(\hat{O}_1^3) \quad , \tag{53}$$

where the $\hat{O}_1^3$ term is $\mathcal{O}(G_F^3)$ and may be neglected. Since the component of $\hat{O}_1$ proportional to h_V^1 is independent of the τ^a, it does not contribute to the commutators in Eq. (53). Hence, we may replace $\hat{O}_1 \to \hat{O}$ in the expression above. Since the $\hat{O}_{2,3}$ are may be of $\mathcal{O}(G_F^0)$, item (1) will generate relevant 3π terms under the general unitary transformation.

From item (2) we obtain

$$\hat{U}^\dagger i V_\mu \hat{U} = -[\hat{O}_1, V_\mu^{(2)}] + \cdots = -[\hat{O}, V_\mu^{(2)}] + \cdots \quad , \quad (54)$$

where $V_\mu^{(2)}$ denotes the 2π terms in V_μ.

Next, consider the contributions from item (3), including the expansion of the $X_\pm^3$. The term proportional to h_V^1 [we neglect the $\mathcal{O}(G_F^3)$] terms is

$$\frac{1}{2} h_V^1 Tr[A_\mu X_+^3] = \frac{2}{3} h_V^1 [\pi^i \pi_i D_\mu \pi^0 - \pi^0 \pi_i D_\mu \pi^i] \quad (55)$$

which does not contain $\tau_\pm, \tau_3$. In order to remove the above term we also need similar terms with $\hat{1}$ structure from Eq. (53)-(54). The commutators never contribute to $\hat{1}$ structure. So the only possibility is the isoscalar piece of $\hat{C}$,

$$D_\mu[b_1 \pi^+ \pi^- \pi^0 + b_2 \pi^0 \pi^0 \pi^0] \quad (56)$$

which is a total derivative of 3π fields, and each term is symmetric under field permutations. However, Eq. (55) does not display such permutation symmetry. In other words, Eqs. (55) and (56) cannot completely cancel each other. Thus, the $3\pi\, h_V^1$ piece will remain under any unitary transformation.

Now consider the h_V^0 term in item (3). Expansion of the A_μ operator in $\hat{U}^\dagger A_\mu \hat{U}$ in Eq. (34) leads to

$$\sim \frac{1}{6} h_V^0 [\pi, [\pi, D_\mu \pi]]$$
$$= \frac{1}{6} h_V^0 \{ \frac{\tau_3}{2} [2\pi^+ \pi^- D_\mu \pi^0 - \pi^0 D_\mu(\pi^+ \pi^-)]$$
$$+ \frac{\tau_+}{\sqrt{2}} [-\pi^+(\pi^+ D_\mu \pi^- - \pi^- D_\mu \pi^+)$$
$$+ \pi^0(\pi^0 D_\mu \pi^+ - \pi^+ D_\mu \pi^0)]$$
$$+ \frac{\tau_-}{\sqrt{2}} [-\pi^-(\pi^- D_\mu \pi^+ - \pi^+ D_\mu \pi^-)$$
$$+ \pi^0(\pi^0 D_\mu \pi^- - \pi^- D_\mu \pi^0)]\} \quad . \quad (57)$$

Finally, from item (4) we obtain for the h_V^0 contribution

$$\sim i h_V^0 [\hat{O}_2, A_\mu] + \mathcal{O}(\hat{O}_1^3) + \cdots$$
$$= i h_V^0 [\hat{O}_2, A_\mu] + \mathcal{O}(G_F^3) + \cdots \quad . \quad (58)$$

Now we require the explicit three π expressions from Eqs. (53, 54,58) [items (1),(2), (4)] in addition to the expression in Eq. (57) [item (3)]. These expressions are linear in the τ_i and $\hat{1}$. For clarity, we first focus on the terms involving τ_3. From Eq. (53) we have

$$\sim D_\mu \hat{O}_3|_{\tau_3} + i h_V^0 [\pi, D_\mu \hat{O}_2]|_{\tau_3}$$
$$- i h_V^0 [D_\mu \pi, \hat{O}_2]|_{\tau_3}$$
$$= D_\mu(b_3 \pi^+ \pi^- \pi^0 + b_4 \pi^0 \pi^0 \pi^0)\tau_3$$
$$+ i\sqrt{2} a_5(h_V^0 + \frac{4}{3} h_V^2)[\pi^+ D_\mu \pi^- - \pi^- D_\mu \pi^+]\pi^0 \tau^3$$
$$+ \sqrt{2} a_6 h_V^0 \pi^+ \pi^- D_\mu \pi^0 \tau_3 \quad (59)$$

where have used the following identity:

$$[\pi, D_\mu \hat{O}_2]|_{\tau_3} = \frac{a_5}{\sqrt{2}}[\pi^+ D_\mu \pi^- - \pi^- D_\mu \pi^+]\pi^0 \tau^3$$
$$- \frac{i a_6}{\sqrt{2}}[2\pi^+ \pi^- D_\mu \pi^0 + \pi^0 D_\mu(\pi^+ \pi^-)]\tau_3 \quad . \quad (60)$$

The contribution from Eq. (54) [item (2)] is

$$- [\hat{O}, V_\mu]|_{\tau_3} = -\frac{1}{4} h_V^0 [2\pi^+ \pi^- D_\mu \pi^0$$
$$- \pi^0 D_\mu(\pi^+ \pi^-)]\tau_3 \quad (61)$$

while from Eq. (57) [item (3)] we obtain

$$+ \frac{1}{12} h_V^0 [2\pi^+ \pi^- D_\mu \pi^0 - \pi^0 D_\mu(\pi^+ \pi^-)]\tau_3 \quad . \quad (62)$$

Finally, Eq. (58) [item (4)] gives

$$i h_V^0 [D_\mu \pi, \hat{O}_2]|_{\tau_3} = -i h_V^0 \frac{a_5}{\sqrt{2}}[\pi^+ D_\mu \pi^-$$
$$- \pi^- D_\mu \pi^+]\pi^0 \tau^3 + h_V^0 \frac{a_6}{\sqrt{2}} \pi^0 D_\mu(\pi^+ \pi^-)\tau^3 \quad . \quad (63)$$

The sum of all four possible sources, *i.e.*, Eq. (59), (61), (62) and (63) yield

$$D_\mu(b_3 \pi^+ \pi^- \pi^0 + b_4 \pi^0 \pi^0 \pi^0)\tau_3$$
$$- \frac{1}{6} h_V^0 [2\pi^+ \pi^- D_\mu \pi^0 - \pi^0 D_\mu(\pi^+ \pi^-)]\tau_3$$
$$+ \sqrt{2} a_6 h_V^0 \pi^+ \pi^- D_\mu \pi^0 \tau_3$$
$$+ \frac{a_6}{\sqrt{2}} h_V^0 \pi^0 D_\mu(\pi^+ \pi^-)\tau_3$$
$$+ i h_V^0 \frac{a_5}{\sqrt{2}}[\pi^+ D_\mu \pi^- - \pi^- D_\mu \pi^+]\pi^0 \tau^3 \quad (64)$$

In order for the transformation (52) to eliminate the 3π vector vertex, the sum in Eq. (64) must vanish. Note the first four lines and the last line of Eq. (64) are, respectively, symmetric and anti-symmetric under the exchange $\pi^+ \leftrightarrow \pi^-$. The symmetric and anti-symmetric terms must vanish separately. The solution is

$$b_3 = -\frac{2}{3} h_V^0$$
$$b_4 = 0$$
$$a_5 = 0$$
$$a_6 = \frac{1}{\sqrt{2}} \quad . \quad (65)$$

Before considering the remaining h_V^0-terms, we observe that the contributions from item (3) involves only expressions involving the pion fields and $\tau_3, \tau_\pm$ multiplied by real coefficients. The operator, $\hat{O}_2$, which contributes via items (1) and (4), only appears in commutators. As a result, the three π terms involving a_{1-5} carry factors of i and, thus, cannot cancel the contributions in (3). Consequently, we set $a_{1-5} = 0$ in what follows.

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Now we consider the terms linear in h_V^0 and τ_+ (the argument for τ_- is identical). The sum of these contributions is

$$(b_6 + ib_8)D_\mu(\pi^0\pi^0\pi^+)$$
$$-\frac{\sqrt{2}}{6}h_V^0\pi^0(\pi^0 D_\mu\pi^+ - \pi^+ D_\mu\pi^0)$$
$$-a_6 h_V^0[\pi^0\pi^0 D_\mu\pi^+ + \pi^0\pi^+ D_\mu\pi^0]$$
$$+\frac{\sqrt{2}}{6}h_V^0\pi^+(\pi^+ D_\mu\pi^- - \pi^- D_\mu\pi^+)$$
$$+(b_5 + ib_7)D_\mu(\pi^+\pi^+\pi^-) \qquad (66)$$

Clearly the last two lines (involving only charged π fields) can never cancel each other. The solution for the first three lines to vanish is

$$a_6 = -\frac{1}{\sqrt{2}}$$
$$b_6 = -\frac{\sqrt{2}}{3}h_V^0$$
$$b_8 = 0 \quad . \qquad (67)$$

Note that the requirements on a_6 in Eqs. (65,67) are not consistent. Thus, it is not possible with the transformation (52) to remove the h_V^0 3π terms from the PV Lagrangian. Moreover, as observed in Ref. [4], Eq. (9) gives the most general PV πNN lagrangian up to one derivative of pion field. There exist no additional PV vector πNN contact interaction terms which start off with three pions. Consequently, the h_V^i cannot not be absorbed as part of other LECs at three pion order.
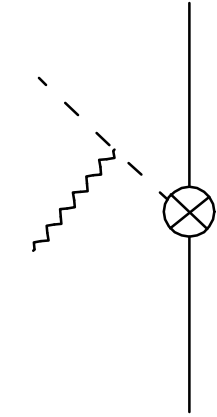
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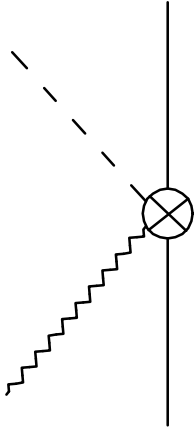[31] A. Manohar and H. Georgi, Nucl. Phys. B 234, 189 (1984).

Figure Captions

Figure 1. The relevant Feyman diagrams for PV π^+ photoproduction. The circle filled with a cross is the PV vertex.
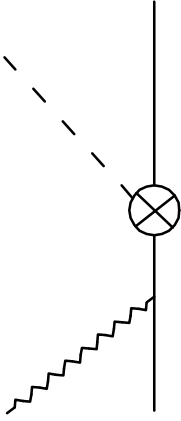
Figure 2. The t-channel ρ-meson exchange diagram used to estimate the PV LEC $\tilde{C}$.
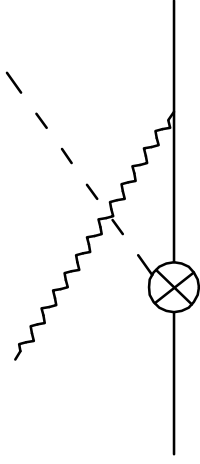
a)   b)   c)   d)